

RESIDUE FORMULA FOR MORITA-FUTAKI-BOTT INVARIANT ON ORBIFOLDS

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ABSTRACT. In this work we prove a residue formula for the Morita-Futaki-Bott invariant with respect to any holomorphic vector field, with isolated (possibly degenerated) singularities in terms of Grothendieck's residues.

Résumé.

Une formule résiduelle pour l'invariant de Morita-Futaki-Bott sur une orbifold

On obtient, en utilisant les résidus de Grothendieck, une formule résiduelle pour l'invariant de Morita-Futaki-Bott par rapport à un champ de vecteurs holomorphe avec singularités isolées, dégénérées ou non.

INTRODUCTION

Let X be a compact complex orbifold of dimension n . That is, X is a complex space endowed with the following property: each point $p \in X$ possesses a neighborhood which is the quotient \tilde{U}/G_p , where \tilde{U} is a complex manifold, say of dimension n , and G_p is a properly discontinuous finite group of automorphisms of \tilde{U} , so that locally we have a quotient map $(\tilde{U}, \tilde{p}) \xrightarrow{\pi_p} (\tilde{U}/G_p, p)$. See [1].

Let $\eta(X)$ be the complex Lie algebra of all holomorphic vector fields of X . Choose any hermitian metric h on X and let ∇ and Θ be the Hermitian connection and the curvature form with respect to h , respectively. Let ξ be a global holomorphic vector field on X and consider the operator

$$L(\xi) := [\xi, \cdot] - \nabla_\xi(\cdot) : T^{1,0}X \longrightarrow T^{1,0}X.$$

Let ϕ be an invariant polynomial of degree $n+k$; the *Futaki-Morita integral invariant* is defined by

$$f_\phi(\xi) = \int_X \bar{\phi} \left(\underbrace{L(\xi), \dots, L(\xi)}_{k \text{ times}}, \underbrace{\frac{i}{2\pi}\Theta, \dots, \frac{i}{2\pi}\Theta}_{n \text{ times}} \right),$$

where $\bar{\phi}$ denotes the polarization of ϕ . Morita and Futaki proved in [7] that the definition of $f_\phi(\xi)$ does not depend on the choice of Hermitian metric h . It is well known that the Futaki-Morita integral invariant can be calculated via a Bott type residue formula for non-degenerated holomorphic vector fields, see [10], [7], [8] and [4] in the orbifold case. In this work we prove a residue formula for holomorphic vector fields with isolated and possibly degenerated singularities in terms of Grothendieck's residues (see [5, Chapter 5]).

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Theorem 1. *Let $\xi \in \eta(X)$ a holomorphic vector field with only isolated singularities, then*

$$\binom{n+k}{n} f_\phi(\xi) = (-1)^k \sum_{p \in \text{Sing}(\xi)} \frac{1}{\#G_p} \text{Res}_{\tilde{p}} \left\{ \frac{\phi(J\tilde{\xi}) d\tilde{z}_1 \wedge \cdots \wedge d\tilde{z}_n}{\tilde{\xi}_1 \cdots \tilde{\xi}_n} \right\},$$

where, given p such that $\xi(p) = 0$ and $(\tilde{U}, \tilde{p}) \xrightarrow{\pi_p} (\tilde{U}/G_p, p)$ denotes the projection: $\tilde{\xi} = \pi_p^* \xi$, $J\tilde{\xi} = \left(\frac{\partial \tilde{\xi}_i}{\partial \tilde{z}_j} \right)_{1 \leq i, j \leq n}$ and $\text{Res}_{\tilde{p}} \left\{ \frac{\phi(J\tilde{\xi}) d\tilde{z}_1 \wedge \cdots \wedge d\tilde{z}_n}{\tilde{\xi}_1 \cdots \tilde{\xi}_n} \right\}$ is Grothendieck's point residue and $(\tilde{z}_1, \dots, \tilde{z}_n)$ is a germ of coordinate system on (\tilde{U}, \tilde{p}) .

We note that such residue can be calculated using Hilbert's Nullstellensatz and Martinelli's integral formula. In fact, if $\tilde{z}_i^{a_i} = \sum_{j=1}^n b_{ij} \tilde{\xi}_j$, then (see [9])

$$(1) \quad \text{Res}_{\tilde{p}} \left\{ \frac{\phi(J\tilde{\xi}) d\tilde{z}_1 \wedge \cdots \wedge d\tilde{z}_n}{\tilde{\xi}_1 \cdots \tilde{\xi}_n} \right\} = \frac{1}{\prod_{i=1}^n (a_i - 1)!} \left(\frac{\partial^n}{\partial \tilde{z}_1^{a_1} \cdots \partial \tilde{z}_n^{a_n}} (\text{Det}(b_{ij}) \phi(J\tilde{\xi})) \right) (\tilde{p}).$$

Moreover, note that if $p \in \text{Sing}(\xi)$ is a not degenerated singularity of ξ then

$$\text{Res}_{\tilde{p}} \left\{ \frac{\phi(J\tilde{\xi}) d\tilde{z}_1 \wedge \cdots \wedge d\tilde{z}_n}{\tilde{\xi}_1 \cdots \tilde{\xi}_n} \right\} = \frac{\phi(J\tilde{\xi}(\tilde{p}))}{\text{Det}(J\tilde{\xi}(\tilde{p}))}.$$

Theorem 1 allows us to calculate the Morita-Futaki invariant for holomorphic vector fields with possible degenerated singularities. For instance, in recent work [6] the Futaki-Bott residue for vector fields with degenerated singularities, on the blowup of Kähler surfaces, was calculated by Li and Shi. Compare the equation (1) with Lemma 3.6 of [6].

Futaki showed in [10] that if X admits a Kähler-Einstein metric then $f_{C_1^{n+1}} \equiv 0$, where $C_1 = \text{Tr}$ denotes the trace, i.e., the first elementary symmetric polynomial. Taking $\phi = C_1^{n+1} = \text{Tr}^{n+1}$, we obtain the following corollary of Theorem 1.

Corollary 2. *Let $\xi \in \eta(X)$ with only isolated singularities, then*

$$f_{C_1^{n+1}}(\xi) = \frac{-1}{(n+1)^2} \sum_{p \in \text{Sing}(\xi)} \frac{1}{\#G_p} \text{Res}_{\tilde{p}} \left\{ \frac{\text{Tr}^{n+1}(J\tilde{\xi}) d\tilde{z}_1 \wedge \cdots \wedge d\tilde{z}_n}{\tilde{\xi}_1 \cdots \tilde{\xi}_n} \right\}.$$

This result generalizes the Proposition 1.2 of [4]. It is well known that projective planes are Kähler-Einstein. However, the non-existence of Kähler-Einstein metrics on singular weighted projective planes was shown in previous works, see for example [12]. As an application of Theorem 1 we will give, in Section 1, a new proof of this fact.

1. NON-EXISTENCE OF KÄHLER-EINSTEIN METRIC ON WEIGHTED PROJECTIVE PLANES

Here we consider weighted complex projective planes with only isolated singularities, which we briefly recall.

Let w_0, w_1, w_2 be positive integers two by two co-primes, set $w := (w_0, w_1, w_2)$ and $|w| := w_0 + w_1 + w_2$. Define an action of \mathbb{C}^* in $\mathbb{C}^3 \setminus \{0\}$ by

$$\begin{aligned} \mathbb{C}^* \times \mathbb{C}^3 \setminus \{0\} &\longrightarrow \mathbb{C}^3 \setminus \{0\} \\ \lambda \cdot (z_0, z_1, z_2) &\longmapsto (\lambda^{w_0} z_0, \lambda^{w_1} z_1, \lambda^{w_2} z_2) \end{aligned}$$

and let $\mathbb{P}_w^2 := \mathbb{C}^3 \setminus \{0\} / \sim$. The weights are chosen to be pairwise co-primes in order to assure a finite number of singularities and to give \mathbb{P}_w^2 the structure of an effective, abelian, compact orbifold of dimension 2. The singular locus is:

$$\text{Sing}(\mathbb{P}_w^2) = \{[1 : 0 : 0]_\omega, [0 : 1 : 0]_\omega, [0 : 0 : 1]_\omega\}.$$

We have the canonical projection

$$\begin{aligned} \pi : \mathbb{C}^3 \setminus \{0\} &\longrightarrow \mathbb{P}_w^2 \\ (z_0, z_1, z_2) &\longmapsto [z_0^{w_0} : z_1^{w_1} : z_2^{w_2}]_w \end{aligned}$$

and the natural map

$$\begin{aligned} \varphi_w : \mathbb{P}^n &\longrightarrow \mathbb{P}_w^n \\ [z_0 : z_1 : z_2] &\longmapsto [z_0^{w_0} : z_1^{w_1} : z_2^{w_2}]_w \end{aligned}$$

of degree $\deg \varphi_w = w_0 w_1 w_2$. The map φ_w is *good* in the sense of [1, section 4.4], which means, among other things, that V-bundles behave well under pullback. It is shown in [11] that there is a line V-bundle $\mathcal{O}_{\mathbb{P}_w^2}(1)$ on \mathbb{P}_w^2 , unique up to isomorphism, such that

$$\varphi_w^* \mathcal{O}_{\mathbb{P}_w^2}(1) \cong \mathcal{O}_{\mathbb{P}^2}(1)$$

and, by naturality, $c_1(\varphi_w^* \mathcal{O}_{\mathbb{P}_w^2}(1)) = c_1(\mathcal{O}_{\mathbb{P}^2}(1)) = \varphi_w^* c_1(\mathcal{O}_{\mathbb{P}_w^2}(1))$, from which we obtain the Chern number

$$[\mathbb{P}_w^2] \frown (c_1(\mathcal{O}_{\mathbb{P}_w^2}(1)))^n = \int_{\mathbb{P}_w^n} (c_1(\mathcal{O}_{\mathbb{P}_w^2}(1)))^2 = \frac{1}{w_0 w_1 w_2}$$

since

$$1 = \int_{\mathbb{P}^2} (c_1(\mathcal{O}_{\mathbb{P}^2}(1)))^2 = \int_{\mathbb{P}^2} \varphi_w^* (c_1(\mathcal{O}_{\mathbb{P}_w^2}(1)))^2 = (\deg \varphi_w) \int_{\mathbb{P}_w^2} (c_1(\mathcal{O}_{\mathbb{P}_w^2}(1)))^2.$$

There exist an Euler type sequence on \mathbb{P}_w^n

$$0 \longrightarrow \underline{\mathbb{C}} \longrightarrow \bigoplus_{i=0}^2 \mathcal{O}_{\mathbb{P}_w^2}(w_i) \longrightarrow T\mathbb{P}_w^2 \longrightarrow 0,$$

where

- (i) $1 \longmapsto (w_0 z_0, w_1 z_1, w_2 z_2)$.
- (ii) $(P_0, P_1, P_2) \longmapsto \pi_* \left(\sum_{i=0}^2 P_i \frac{\partial}{\partial z_i} \right)$.

It is well known that the non-singular weighted projective planes admit Kähler-Einstein metrics. On the other side, singular weighted projective spaces do not admit Kähler-Einstein metrics, see [12]. We give a simple proof of the non-existence of Kähler-Einstein metrics on singular \mathbb{P}_ω^2 by using Corollary 2.

Theorem 3. The singular weighted projective space \mathbb{P}_ω^2 does not admit any Kähler-Einstein metric

Proof. Choose $a_0, a_1, a_2 \in \mathbb{C}^*$ such that $a_i w_j \neq a_j w_i$, for all $i \neq j$. Suppose, without loss of generality, that $1 \leq w_0 \leq w_2 < w_1$. Consider the holomorphic vector field on \mathbb{P}_ω^2 given by

$$\xi_a = \sum_{k=0}^2 a_k Z_k \frac{\partial}{\partial Z_k} \in H^0(\mathbb{P}_\omega^2, T\mathbb{P}_\omega^2),$$

where (Z_0, Z_1, Z_3) denotes the homogeneous coordinate system.

The local expression of ξ over $U_i = \{[Z_0 : Z_1 : Z_3] \in \mathbb{P}^2; Z_i \neq 0\}$ is given by

$$\xi_a|_{U_i} = \sum_{\substack{k=0 \\ k \neq i}}^2 \left(a_k - a_i \frac{w_k}{w_i} \right) Z_k \frac{\partial}{\partial Z_k},$$

Therefore, the singular set $Sing(\xi|_{U_i})$ is reduced to $\{0\}$ and it is nondegenerate. In general

$$Sing(\xi_a) = \{[1 : 0 : 0]_\omega, [0 : 1 : 0]_\omega, [0 : 0 : 1]_\omega\} = Sing(\mathbb{P}_\omega^2).$$

It follows from Corollary 2 that

$$f(\xi_a) = \frac{-1}{3^2} \sum_{i=0}^2 \frac{1}{w_i^2} \frac{(\sum_{k \neq i} (a_k w_i - a_i w_k))^3}{\prod_{k \neq i} (a_k w_i - a_i w_k)}.$$

Thus

$$\begin{aligned} \zeta(a_0, a_1, a_2) &= -3^2 w_0^2 w_1^2 w_2^2 \prod_{0 \leq i < j \leq 2} (a_i w_j - a_j w_i) f(\xi_a) = \\ &= (3w_1^5 w_2^2 w_0 - 3w_1^4 w_2^3 w_0 + 3w_1^3 w_2^4 w_0 + 3w_1^2 w_2^5 w_0 - 3w_0^4 w_2^2 w_1^2 + 3w_0^3 w_2^3 w_1^2 + 6w_0^2 w_2^4 w_1^2 + \\ &+ 3w_0^4 w_1^2 w_2^2 - 3w_0^3 w_1^3 w_2^2 - 6w_0^2 w_1^4 w_2^2) \cdot a_1 a_2 a_0^2 + \dots \end{aligned}$$

is a homogeneous polynomial of degree 4 in the variables a_0, a_1, a_2 . Suppose by contradiction that $\zeta(a_0, a_1, a_2) \equiv 0$. In particular the coefficient of the monomial $a_0^2 a_1 a_2$ is zero. Thus, we have the following equation

$$w_2(w_1 w_2 + w_2^2 + w_0^2 + 2w_0 w_2) = w_1(w_1 w_2 + w_1^2 + w_0^2 + 2w_0 w_1).$$

This contradicts $1 \leq w_0 \leq w_2 < w_1$. Thus the non-vanishing of $\zeta(a_0, a_1, a_2)$ implies that $f(\xi_a)$ is not zero. Therefore \mathbb{P}_ω^2 does not admit Kähler-Einstein metrics. \square

2. PROOF OF THEOREM 1

For the proof we will use Bott-Chern's transgression method, see [2] and [3].

Let p_1, \dots, p_m be the zeros of ξ . Let $\{U_\beta\}$ be an open cover orbifold of X ($\varphi_\beta : \tilde{U}_\beta \rightarrow U_\beta \subset X$ coordinate map). Suppose that $\{U_\beta\}$ is a trivializing neighborhood for the holomorphic tangent orbibundle TX (see [1, section 2.3]) of X and that we have disjoint neighborhoods coordinates U_α with $p_\alpha \in U_\alpha$ and $p_\alpha \notin U_\beta$ if $\alpha \neq \beta$. On each \tilde{U}_α , take local coordinates $\tilde{z}^\alpha = (\tilde{z}_1^\alpha, \dots, \tilde{z}_n^\alpha)$ and the holomorphic frame $\{\frac{\partial}{\partial \tilde{z}_1^\alpha}, \dots, \frac{\partial}{\partial \tilde{z}_n^\alpha}\}$ of TX . Thus, we have a local representation

$$\tilde{\xi}^\alpha = \sum \tilde{\xi}_i^\alpha \frac{\partial}{\partial \tilde{z}_i^\alpha},$$

where $\tilde{\xi}_i^\alpha$ are holomorphic functions in \tilde{U}_α , $1 \leq i \leq n$. Let \tilde{h}_α the Hermitian metric in \tilde{U}_α defined by $\langle \partial/\partial \tilde{z}_i^\alpha, \partial/\partial \tilde{z}_j^\alpha \rangle = \delta_j^i$. Also consider $\tilde{U}'_\alpha \subset \tilde{U}_\alpha$ e $U'_\alpha = \varphi_\alpha(\tilde{U}'_\alpha)$ for each α . Take a Hermitian metric h_0 in any $X \setminus \cup_\alpha \{p_\alpha\}$ and $\{\rho_0, \rho_\alpha\}$ a partition of unity subordinate to the cover $\{X \setminus \cup_\alpha \overline{U}'_\alpha, U_\alpha\}_\alpha$. Define a Hermitian metric $h = \rho_0 h_0 + \sum \rho_\alpha h_\alpha$ in X . Then we have that for every α , the metric curvature $\Theta \equiv 0$ in U'_α .

Consider the matrix of the metric connection ∇ in the open \tilde{U}^β given by $\theta^\beta = (\sum_k \Gamma_{ik}^{\beta j} d\tilde{z}_k^\beta)$. The local expression of $L(\xi)$ is given by $\tilde{E}^\beta = (\tilde{E}_{ij}^\beta)$ such that

$$\tilde{E}_{ij}^\beta = -\frac{\partial \tilde{\xi}_i^\beta}{\partial \tilde{z}_j^\beta} - \sum_s \Gamma_{js}^{\beta i} \tilde{\xi}_s^\beta,$$

see [2] and [5]. We indicate by $\mathcal{A}^{p,q}(X)$ the vector space of complex-valued $(p+q)$ -forms on X of type (p, q) . Define

$$\phi_r := \binom{n+k}{r} \bar{\phi}(\underbrace{E, \dots, E}_{n+k-r}, \underbrace{\Theta, \dots, \Theta}_r) \in \mathcal{A}^{r,r}(X) \quad r = 0, \dots, n.$$

Let $\omega \in \mathcal{A}^{1,0}(X)$ in $X \setminus \text{Sing}(\xi)$, with $\omega(\xi) = 1$. Following the Bott's idea (see [2]), it is sufficient to show that there exists ψ such that $i(\xi)(\bar{\partial}\psi + \phi_n) = 0$ on $X \setminus \text{Sing}(\xi)$. We take $\psi = \sum_{r=0}^{n-1} \psi_r$ such that

$$\psi_r = \omega \wedge (\bar{\partial}\omega)^{n-r-1} \wedge \phi_r \in \mathcal{A}^{n,n-1}(X) \quad r = 0, \dots, n-1.$$

The following formulas hold (see [2] or [5]) :

- a) $\bar{\partial}\Theta = 0, \bar{\partial}E = i(\xi)\Theta$
- b) $\bar{\partial}\phi_r = i(\xi)\phi_{r+1} \quad r = 0, \dots, n-1$
- c) $i(\xi)\bar{\partial}\omega = 0$

Let us prove b) : since $\bar{\partial}\Theta = 0$ and $\bar{\partial}E = i(\xi)\Theta$, we have

$$\bar{\partial}\phi_r = \binom{n+k}{r} \sum_{i=1}^{n+k-r} \bar{\phi}(E, \dots, i(\xi)\Theta, \dots, E, \Theta, \dots, \Theta) = i(\xi)\phi_{r+1}.$$

Therefore, a), b) and c) implies that on $X \setminus \text{Sing}(\xi)$ we get

$$i(\xi)(\bar{\partial}\psi + \phi_n) = 0.$$

Therefore $d\psi = \bar{\partial}\psi = -\phi_n$ on $X \setminus \text{Sing}(\xi)$. Thus, by Satake-Stokes Theorem we have

$$\begin{aligned} \binom{n+k}{n} f_\phi(\xi) &= \left(\frac{i}{2\pi}\right)^n \int_X \phi_n = \left(\frac{i}{2\pi}\right)^n \lim_{\epsilon \rightarrow 0} \int_{X \setminus \cup_\alpha B_\epsilon(p_\alpha)} \phi_n \\ (2) \quad &= -\left(\frac{i}{2\pi}\right)^n \lim_{\epsilon \rightarrow 0} \int_{X \setminus \cup_\alpha B_\epsilon(p_\alpha)} d\psi = \left(\frac{i}{2\pi}\right)^n \lim_{\epsilon \rightarrow 0} \sum_\alpha \int_{\partial B_\epsilon(p_\alpha)} \psi^\alpha, \end{aligned}$$

where is $B_\epsilon(p_\alpha) = B_\epsilon(\tilde{p}_\alpha)/G_{p_\alpha}$ and $B_\epsilon(\tilde{p}_\alpha)$ is an Euclidean ball centered at \tilde{p}_α such that $\overline{B_\epsilon(\tilde{p}_\alpha)} \subset U'_\alpha$. Since our metric is Euclidean in $B_\epsilon(\tilde{p}_\alpha)$, its connection is zero and

$$\tilde{E}_{ij}^\alpha = -\frac{\partial \tilde{\xi}_i^\alpha}{\partial \tilde{z}_j^\alpha}.$$

Now, by our choice of metric, Θ and hence ϕ_r , for $r > 0$, vanishes identically in $B_\epsilon(\tilde{p}_\alpha)$. Then, we have

$$\tilde{\psi}^\alpha = \tilde{\psi}_0^\alpha = \omega \wedge (\bar{\partial}\omega)^{n-1} \phi(\tilde{E}^\alpha) = (-1)^{n+k} \omega \wedge (\bar{\partial}\omega)^{n-1} \phi(J\tilde{\xi}^\alpha)$$

on $B_\epsilon(\tilde{p}_\alpha)$. Therefore

$$(3) \quad \tilde{\psi}^\alpha = (-1)^k \omega \wedge (\bar{\partial}\omega)^{n-1} \phi(J\tilde{\xi}^\alpha).$$

Consider the map $\Phi : \mathbb{C}^n \rightarrow \mathbb{C}^{2n}$ given by $\Phi(\tilde{z}) = (\tilde{z} + \tilde{\xi}(\tilde{z}), \tilde{z})$. There is a $(2n, 2n - 1)$ closed form β_n in $\mathbb{C}^{2n} \setminus \{0\}$ (the Bochner-Martinelli kernel) such that

$$(4) \quad \Phi^* \beta_n = \left(\frac{i}{2\pi} \right)^n \omega \wedge (\bar{\partial} \omega)^{n-1}.$$

Finally, if we substitute (3) and (4) in (2), and by using Martinelli's formula ([5, pg. 655])

$$\int_{\partial B_\epsilon(\tilde{p}_\alpha)} \phi(J\tilde{\xi}^\alpha) \Phi^* \beta_n = \text{Res}_{\tilde{p}_\alpha} \left\{ \frac{\phi(J\tilde{\xi}^\alpha) d\tilde{z}_1 \wedge \cdots \wedge d\tilde{z}_n}{\tilde{\xi}_1 \cdots \tilde{\xi}_n} \right\}$$

we obtain

$$\binom{n+k}{n} f_\phi(\xi) = (-1)^k \sum_{\alpha} \frac{1}{\#G_{p_\alpha}} \text{Res}_{\tilde{p}_\alpha} \left\{ \frac{\phi(J\tilde{\xi}^\alpha) d\tilde{z}_1 \wedge \cdots \wedge d\tilde{z}_n}{\tilde{\xi}_1 \cdots \tilde{\xi}_n} \right\}.$$

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REFERENCES

- [1] A. ADEM, J. LEIDA, Y. RUAN, *Orbifolds and String Topology*, Cambridge University Press, ISBN-10 0-511-28288-5, 2007.
- [2] R. BOTT, *Vector fields and characteristic numbers*, Michigan Math. Jour. Vol. 14, (1967) 231-244.
- [3] S. S. CHERN, *Meromorphic vector fields and characteristic numbers*, Selected Papers, Springer-Verlag, New York, 1978, 435-443.
- [4] W. DING, G. TIAN, *Kähler-Einstein metrics and the generalized Futaki invariant*, Invent. Math. 110, 315-335 (1992).
- [5] P. GRIFFITHS, J. HARRIS, *Principles of algebraic geometry*, Wiley, 1978.
- [6] H. LI, Y. SHI, *The Futaki invariant on the blowup of Kähler surfaces*. Int. Math. Res. Notices (2014) doi: 10.1093/imrn/rnt351.
- [7] A. FUTAKI, S. MORITA, *Invariant Polynomials on Compact Complex Manifolds*, Proc. Japan Acad., 60, Ser. A 1984.
- [8] A. FUTAKI, S. MORITA : *Invariant polynomials of the automorphism group of a compact complex manifold*, J. Differential Geom. 21 (1985), 135-142.
- [9] F. NORGUET, *Fonctions de plusieurs variables complexes*, Lecture notes in Mathematics, 409 (1974) 1-97.
- [10] A. FUTAKI, *An Obstruction to the Existence of Einstein Kähler Metrics*, Invent. Math. 73, 437-443 (1983).
- [11] É MANN, *Cohomologie quantique orbifolde des espaces projectifs à poids*, J. Algebraic Geom. 17 (2008), 137-166.
- [12] J. A. VIACLOVSKY, *Einstein metrics and Yamabe invariants of weighted projective spaces*. Tohoku Math. J. (2) Vol. 65, Number 2 (2013), 297-311.

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